

Покажите, что последовательность $c_n = (1 + 1/n)^{n+1}$ убывающая

Указание:

Как связаны последовательности $c_n = (1 + 1/n)^{n+1}$ и $b_n = (1 - 1/n)^n$:

$$c_n = (1 + 1/n)^{n+1} = ((n+1)/n)^{n+1} = 1/(n/(n+1))^{n+1} = 1/((n+1-1)/(n+1))^{n+1} = 1/(1 - 1/(n+1))^{n+1} = 1/b_{n+1}$$

$$c_n = \left(1 + \frac{1}{n}\right)^{n+1} \downarrow ?$$

$$b_n = \left(1 - \frac{1}{n}\right)^n \nearrow$$

$$b_{n+1} = \left(1 - \frac{1}{n+1}\right)^{n+1} \nearrow \quad \frac{1}{b_n} \searrow$$

$$c_n = \frac{1}{b_{n+1}} \searrow$$

$$\begin{aligned} c_n &= \left(1 + \frac{1}{n}\right)^{n+1} = \left(\frac{n+1}{n}\right)^{n+1} = \frac{1}{\left(\frac{n}{n+1}\right)^{n+1}} = \frac{1}{\left(\frac{n+1-1}{n+1}\right)^{n+1}} \\ &= \frac{1}{\left(1 - \frac{1}{n+1}\right)^{n+1}} = \frac{1}{b_{n+1}} \end{aligned}$$

$$(1 + 1/n)^{n+1} \downarrow$$